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LETTER TO THE EDITOR

Generation of three-dimensional graph state with Josephson charge qubits

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Abstract
On the basis of generations of one-dimensional and two-dimensional graph states, we generate a three-dimensional N^3-qubit graph state based on Josephson charge qubits. Since any two charge qubits can be selectively and effectively coupled by a common inductance, the controlled phase transform between any two-qubit states can be performed. Accordingly, we can generate arbitrary multi-qubit graph states corresponding to arbitrary shape graphs, which meet the expectations of various quantum information processing schemes. All the devices in the scheme are well within current technology. It is a simple, scalable and feasible scheme for the generation of various graph states based on Josephson charge qubits.

Entanglement [1] can serve as a basic ingredient in the course of quantum information processing. In achieving the task of quantum communication, the entanglement is a medium for transferring quantum information. Owing to entanglement, quantum computers have potentially superior computing power over their classical counterparts. Graph states [2–4] are a family of multi-qubit states. Many well-known states, such as Greenberger–Horne–Zeilinger (GHZ) [5] states and cluster states [6], can be generated from the graph states. In quantum error correcting codes [7, 8] and in one-way quantum computing [9, 10], some of the graph states are used as resources. For every non-trivial graph state it is possible to construct three-setting Bell inequalities which are maximally violated only by this state [11, 12].

The concept of a graph is the basis of a graph state. A graph $G = (V, E)$ comprises two classes of element, i.e., vertices V and edges E. Each graph can be represented by a diagram in a plane, where each vertex is represented by a point and each edge E by an arc joining two not necessarily distinct vertices [3]. For the graph states, vertices V correspond

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to qubits of physical systems and edges represent interactions of qubits. The state vector $|\Psi\rangle = |+\rangle^{\otimes V} = ((|0\rangle + |1\rangle)/\sqrt{2})^{\otimes V}$ is referred to as the graph state vector of the empty graph. The state vector of the graph state containing edges is described as

$$|G\rangle = \prod_{(i,j\in E)} U^{(i,j)}|\Psi\rangle = \prod_{(i,j\in E)} U^{(i,j)}((|0\rangle + |1\rangle)/\sqrt{2})^{\otimes V} \tag{1}$$

with $U^{(i,j)} = (I^{(i)} \otimes I^{(j)} + \sigma_z^{(i)} \otimes I^{(j)} + I^{(i)} \otimes \sigma_z^{(j)} - \sigma_z^{(i)} \otimes \sigma_z^{(j)})/2$ corresponding to a controlled phase-gate between qubits labelled i and j, described by Pauli matrices. As mentioned above, the graph states have special characteristics and practical applications, so the preparation of the graph states has become the focus of research. Clark *et al* [7] present a scheme that allows arbitrary graph states to be efficiently created in a linear quantum register via an auxiliary entangling bus. Benjamin *et al* [13] present a scheme that creates graph states by simple three-level systems in separate cavities. Bodiya *et al* [14] propose a scheme for efficient construction of graph states using realistic linear optics, an imperfect photon source and single-photon detectors.

Recently, much attention has been attracted to the quantum computer, which works on the fundamental quantum mechanical principle. Quantum computers can solve some problems exponentially faster than classical computers. For realizing quantum computing, some physical systems, such as nuclear magnetic resonance [15], trapped irons [16], cavity quantum electrodynamics (QED) [17], and optical systems [18], have been proposed. These systems have the advantage of high quantum coherence, but cannot be integrated easily to form large-scale circuits. Because of large-scale integration and relatively high quantum coherence, the Josephson charge qubit [19–21] and the flux qubit [22, 23], which are based on the macroscopic quantum effects in low-capacitance Josephson junction circuits [24, 25], are the promising candidates for quantum computing. As is well known, the graph states are mainly applied to quantum computing. Accordingly, generation of the graph states by Josephson charge and flux qubits is of great importance. In this paper, we propose a scheme for the generation of the graph states using Josephson charge qubits. This scheme is simple and easily manipulated, because any two charge qubits can be selectively and effectively coupled by a common inductance. More manipulations can be realized before decoherence sets in. All of the devices in the scheme are well within current technology. It is a simple, scalable and feasible scheme for the generation of various graph states based on Josephson charge qubits.

This letter is organized as follows. First, we introduce the Josephson charge-qubit structure and the Hamiltonian of the system. Second, we explain how to implement the controlled phase-gate. Third, we illustrate the generation of the arbitrary multi-qubit graph states corresponding to arbitrary shape graphs. Fourth, we give necessary discussions for the feasibility of our scheme. Finally, the conclusions are given.

Since the earliest Josephson charge qubit scheme [19] was proposed, a series of improved schemes [20, 26] has been explored. Here, we concern ourselves with the architecture of the Josephson charge qubit in reference [26], which is the first efficient scalable quantum computing (QC) architecture. The Josephson charge qubit structure is shown in figure 1. It consists of N Cooper-pair boxes (CPBs) coupled by a common superconducting inductance L. For the kth Cooper-pair box, a superconducting island with charge $Q_k = 2en_k$ is weakly coupled by two symmetric direct current superconducting quantum interference devices (dc SQUIDs) and biased by an applied voltage through a gate capacitance C_k. Assume that the two symmetric dc SQUIDs are identical and all Josephson junctions in them have Josephson coupling energy E_{Jk}^0 and capacitance C_{Jk}. The self-inductance effects of each SQUID loop are usually neglected because of the very small size (1 μm) of the loop. Each SQUID pierced by a magnetic flux Φ_{Xk} provides an effective coupling energy $-E_{Jk}(\Phi_{Xk})\cos\phi_{kA(B)}$, with

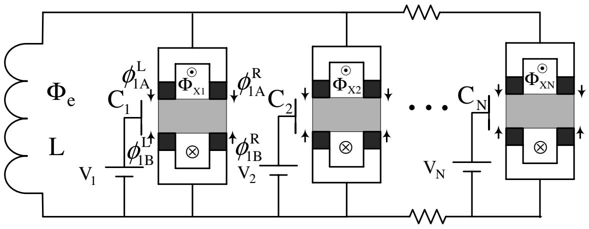

Figure 1. Josephson charge-qubit structure. Each CBP is configured both in the charging regime $E_{ck} \gg E^0_{Jk}$ and at low temperatures $k_B T \ll E_{ck}$. Furthermore, the superconducting gap Δ is larger than E_{ck} so that quasiparticle tunnelling is suppressed in the system.

$E_{Jk}(\Phi_{Xk}) = 2E^0_{Jk}\cos(\pi\Phi_{Xk}/\Phi_0)$, and the flux quantum $\Phi_0 = h/2e$. The effective phase drop $\phi_{kA(B)}$, with subscript $A(B)$ labelling the SQUID above (below) the island, equals the average value, $[\phi^{\mathrm{L}}_{kA(B)} + \phi^{\mathrm{R}}_{kA(B)}]/2$, of the phase drops across the two Josephson junctions in the dc SQUID, with superscript L(R) denoting the left (right) Josephson junction.

For any given Cooper-pair box, say i, when $\Phi_{Xk} = \frac{1}{2}\Phi_0$ and $V_{Xk} = (2n_k + 1)e/c_k$ for all boxes except $k = i$, the inductance L connects only the ith Cooper-pair box to form a superconducting loop. In the spin-$\frac{1}{2}$ representation, based on charge states $|0\rangle = |n_i\rangle$ and $|1\rangle = |n_{i+1}\rangle$, the reduced Hamiltonian of the system becomes [26]

$$H = \varepsilon_i(V_{Xi})\sigma_z^{(i)} - \bar{E}_{Ji}(\Phi_{Xi}, \Phi_{\mathrm{e}}, L)\sigma_x^{(i)}, \tag{2}$$

where $\varepsilon_i(V_{Xi})$ is controlled by the gate voltage V_{Xi}, while the intrabit coupling $\bar{E}_{Ji}(\Phi_{Xi}, \Phi_{\mathrm{e}}, L)$ depends on inductance L, the applied external flux Φ_{e} through the common inductance and the local flux Φ_{Xi} through the two SQUID loops of the ith Cooper-pair box. By controlling Φ_{Xk} and V_{Xk}, the operations of Pauli matrices $\sigma_z^{(i)}$ and $\sigma_x^{(i)}$ are achieved. Thus, any single-qubit operations are realized by utilizing equation (1).

To manipulate many-qubit states, say i and j, we configure $\Phi_{Xk} = \frac{1}{2}\Phi_0$ and $V_{Xk} = (2n_k + 1)e/c_k$ for all boxes except $k = i$ and j. In this case, the inductance L is only shared by the Cooper-pair boxes i and j to form superconducting loops. The Hamiltonian of the system can be reduced to [26, 27]

$$H = \sum_{k=i,j}[\varepsilon_k(V_{Xk})\sigma_z^{(k)} - \bar{E}_{Jk}\sigma_x^{(k)}] + \Pi_{ij}\sigma_x^{(i)}\sigma_x^{(j)}, \tag{3}$$

where the interbit coupling Π_{ij} depends on both the external flux Φ_{e} through the inductance L, the local fluxes Φ_{Xi} and Φ_{Xj} through the SQUID loops. In equation (2), if we choose $V_{Xk} = (2n_k + 1)e/c_k$, the Hamiltonian of the system can be reduced to

$$H = -\bar{E}_{Ji}\sigma_x^{(i)} - \bar{E}_{Jj}\sigma_x^{(j)} + \Pi_{ij}\sigma_x^{(i)}\sigma_x^{(j)}. \tag{4}$$

For simplicity of calculation, we assume $\bar{E}_{Ji} = \bar{E}_{Jj} = \Pi_{ij} = \frac{-\pi\hbar}{4\tau}$ (τ is a given period of time), which can be obtained by suitably choosing parameters. Thus equation (3) becomes

$$H = \frac{-\pi\hbar}{4\tau}(-\sigma_x^{(i)} - \sigma_x^{(j)} + \sigma_x^{(i)}\sigma_x^{(j)}). \tag{5}$$

Below, we discuss problems on the basis $\{|+\rangle = \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle), |-\rangle = \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\}$. According to Hamiltonian H of equation (5), we can obtain the following evolutions:

$$|{++}\rangle_{ij} \rightarrow \mathrm{e}^{-\mathrm{i}\pi t/4\tau}|{++}\rangle_{ij}, \tag{6a}$$

$$|{+-}\rangle_{ij} \rightarrow \mathrm{e}^{-\mathrm{i}\pi t/4\tau}|{+-}\rangle_{ij}, \tag{6b}$$

$$|{-+}\rangle_{ij} \rightarrow \mathrm{e}^{-\mathrm{i}\pi t/4\tau}|{-+}\rangle_{ij}, \tag{6c}$$

$$|{--}\rangle_{ij} \rightarrow \mathrm{e}^{\mathrm{i}3\pi t/4\tau}|{--}\rangle_{ij}. \tag{6d}$$

If we choose $t = \tau$, which can be achieved by choosing the switching time, and perform a single-qubit operation $U = \mathrm{e}^{\mathrm{i}\pi/4}$, we can obtain

$$|{+}{+}\rangle_{ij} \rightarrow |{+}{+}\rangle_{ij}, \tag{7a}$$

$$|{+}{-}\rangle_{ij} \rightarrow |{+}{-}\rangle_{ij}, \tag{7b}$$

$$|{-}{+}\rangle_{ij} \rightarrow |{-}{+}\rangle_{ij}, \tag{7c}$$

$$|{-}{-}\rangle_{ij} \rightarrow -|{-}{-}\rangle_{ij}. \tag{7d}$$

Equations (7) have actually realized the operation of a controlled phase gate. Any two charge qubits can be selectively and effectively coupled by a common inductance, so the controlled phase transform between any two-qubit states is performed. It is very important for the following generation of arbitrary multi-qubit graph states corresponding to arbitrary shape graphs.

Under the basis of $\{|+\rangle, |-\rangle\}$, the state vector of the graph state containing edges is described as

$$|G\rangle = \prod_{(i,j\in E)} U'^{(i,j)}((|+\rangle + |-\rangle)/\sqrt{2})^{\otimes V}, \tag{8}$$

where $U'^{(i,j)}$ is a controlled phase-gate for the basis of $\{|+\rangle, |-\rangle\}$. Our goal is generating a three-dimensional N^3-qubit graph states corresponding to a three-dimensional graph. The work of generating three-dimensional graph state divides into the following three steps.

Step 1: first, we prepare all N charge qubits in the states of $|+\rangle$, which is the graph state of an empty graph. Next, we perform $N-1$ controlled phase transforms between adjacent charge qubits in the x-axis direction as shown in figure 2(a). Thus we obtain a one-dimensional graph state corresponding to the right graph of the figure 2(a).

Step 2: first, we prepare N graph states of one dimension, which are the N-qubit graph states corresponding to the right graph in the figure 2(a). Next, on the basis of the left graph of the figure 2(b), we perform $N(N-1)$ controlled phase transforms between adjacent charge qubits in the y-axis direction as shown in figure 2(b). Thus we obtain a two-dimensional graph state corresponding to the right graph in the figure 2(b).

Step 3: first, we prepare N graph states of two dimensions, which are the N^2-qubit graph states corresponding to the right graph in the figure 2(b). Next, on the basis of the left graph of the figure 2(c), we perform $N^2(N-1)$ controlled phase transforms between adjacent charge qubits in the z-axis direction as shown in figure 2(c). Thus we obtain a three-dimensional graph state, which is an N^3-qubit graph state corresponding to the right graph in figure 2(c).

It is worth adding that our scheme can generalize to generate arbitrary multi-qubit graph states corresponding to arbitrary shape graphs, which meet the expectations of various quantum information processing schemes.

Below, we briefly discuss the experimental feasibility of the current scheme. For the charge qubit used in our scheme, the typical experimental switching time $\tau^{(1)}$ during a single-bit operation is about 0.1 ns [26]. The inductance L used in our proposal is about 30 nH, which is experimentally accessible. In the earlier design [20], the inductance L is about 3.6 μH, which is difficult to make at nanometre scales. Another improved design [24] greatly reduces the inductance L to $\sim$120 nH, which is about four times larger than the one used in our scheme. The fluctuations of voltage source and fluxes result in decoherence for all charge qubits. The gate voltage fluctuation plays the dominant role in producing decoherence. The estimated dephasing time is $\tau_4 \sim 10^{-4}$ s [24], which allows in principle 10^6 coherent single-bit manipulations. Owing to using the probe junction, the phase coherence time is only about 2 ns [28, 29]. In this setup, background charge fluctuations and the probe–junction measurement may be two of the major factors in producing decoherences [26]. The charge fluctuations are principally

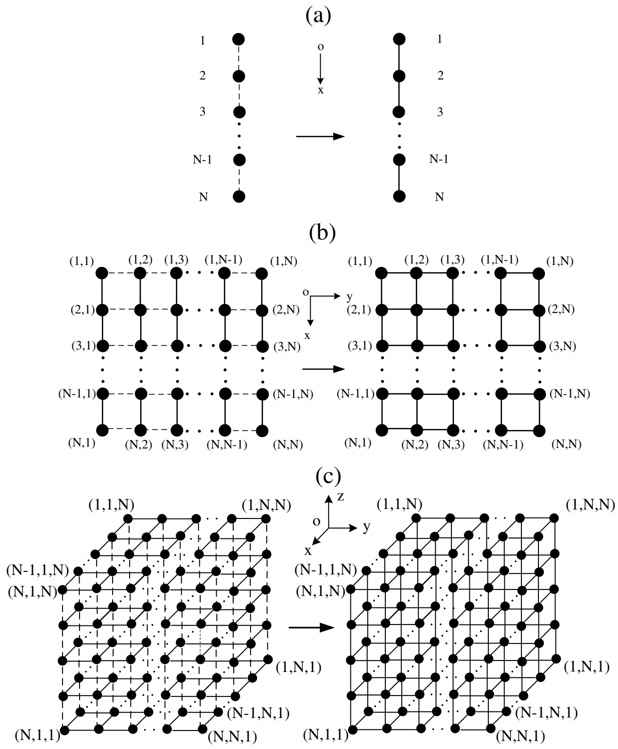

Figure 2. The dashed lines denote that the interactions between qubits have not taken place. The real lines denote that the interactions between qubits have been completed. (a) Generation of N-qubit graph state from the graph state of an empty graph to a one-dimensional graph state. (b) Generation of the graph state from one-dimensional graph states to two-dimensional graph state. (c) Generation of the graph state from two-dimensional graph states to three-dimensional graph state.

only in the low-frequency region and they can be reduced by the echo technique [28] and by controlling the gate voltage to the degeneracy point, but an effective technique for suppressing charge fluctuations still needs to be explored. According to the discussion above, all the devices in our scheme are achievable by current technology.

In summary, we have investigated a simple scheme for generating the graph states based on Josephson charge qubits. First, we generate a one-dimensional N-qubit graph state from a graph state corresponding to an empty graph. Next, on the basis of the first step, we generate a two-dimensional N^2-qubit graph state. Finally, on the basis of the second step, we generate a three-dimensional N^3-qubit graph state. Since any two charge qubits can be selectively and effectively coupled by a common inductance, the controlled phase transform between any two-qubit states is performed. Accordingly, we can generate arbitrary multi-qubit graph states corresponding to arbitrary shape graphs, which meet the expectations of various quantum information processing schemes. The architecture of our proposal is achievable by current scalable microfabrication techniques. More manipulations can be realized before decoherence sets in. All the devices in the scheme are well within current technology. It is a simple, scalable and feasible scheme for the generation of various graph states based on Josephson charge qubits.

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